Ejercicio 1 del método de bisección. La ecuación estudiada por Leonardo de Pisa.

Es importante la terminología.

> $f := x \rightarrow x^3 + 2 \cdot x^2 + 10 \cdot x - 20;$

$$f := x \rightarrow x^3 + 2\,x^2 + 10\,x - 20 \quad \textbf{(1)}$$

> $f(1);$

$$-7 \quad \textbf{(2)}$$

> $f(2);$

$$16 \quad \textbf{(3)}$$

>

Métodos de Maple.

Hallamos las raíces.

>

> $fsolve(f);$

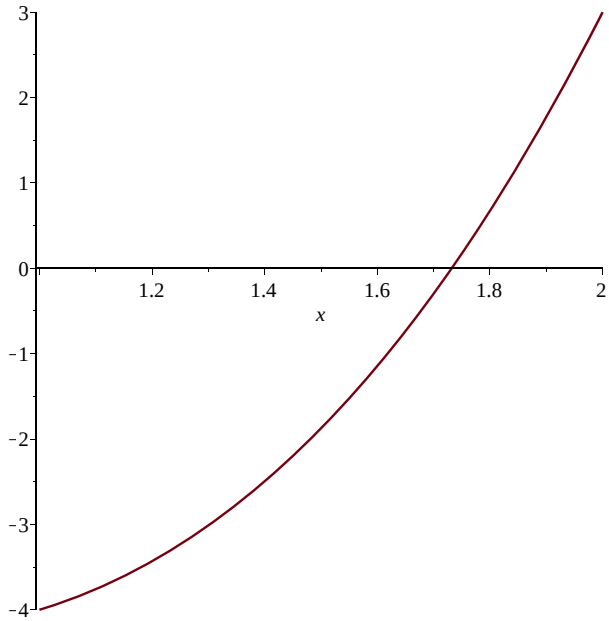
$$1.368808108 \quad \textbf{(4)}$$

> $solve(f(x));$

$$\frac{1}{3}\left(352+6\sqrt{3930}\right)^{1/3} - \frac{26}{3\left(352+6\sqrt{3930}\right)^{1/3}} - \frac{2}{3}, -\frac{1}{6}\left(352+6\sqrt{3930}\right)^{1/3} + \frac{13}{3\left(352+6\sqrt{3930}\right)^{1/3}} - \frac{2}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(352+6\sqrt{3930}\right)^{1/3} + \frac{26}{3\left(352+6\sqrt{3930}\right)^{1/3}}\right), -\frac{1}{6}\left(352+6\sqrt{3930}\right)^{1/3} + \frac{13}{3\left(352+6\sqrt{3930}\right)^{1/3}} - \frac{2}{3} - \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(352+6\sqrt{3930}\right)^{1/3} + \frac{26}{3\left(352+6\sqrt{3930}\right)^{1/3}}\right) \quad \textbf{(5)}$$

>

> $plot(f(x), x = 1..2);$



>

>

> $a := 1; b := 2; n := 7; h := \frac{(b-a)}{2^{n-1}};$

$$a := 1$$
$$b := 2$$
$$n := 7$$
$$h := \frac{1}{64} \quad \textbf{(6)}$$

>

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 1.015625000, −6.733150482
2, 1.031250000, −6.463836670
3, 1.046875000, −6.192035675
4, 1.062500000, −5.917724609
5, 1.078125000, −5.640880585
6, 1.093750000, −5.361480713

7, 1.109375000, −5.079502106
8, 1.125000000, −4.794921875
9, 1.140625000, −4.507717133
10, 1.156250000, −4.217864990
11, 1.171875000, −3.925342560
12, 1.187500000, −3.630126953
13, 1.203125000, −3.332195282
14, 1.218750000, −3.031524658
15, 1.234375000, −2.728092194
16, 1.250000000, −2.421875000
17, 1.265625000, −2.112850189
18, 1.281250000, −1.800994873
19, 1.296875000, −1.486286163
20, 1.312500000, −1.168701172
21, 1.328125000, −0.8482170105
22, 1.343750000, −0.5248107910
23, 1.359375000, −0.1984596252
24, 1.375000000, 0.1308593750
25, 1.390625000, 0.4631690979
26, 1.406250000, 0.7984924316
27, 1.421875000, 1.136852264
28, 1.437500000, 1.478271484
29, 1.453125000, 1.822772980
30, 1.468750000, 2.170379639
31, 1.484375000, 2.521114349
32, 1.500000000, 2.875000000
33, 1.515625000, 3.232059479
34, 1.531250000, 3.592315674
35, 1.546875000, 3.955791473
36, 1.562500000, 4.322509766
37, 1.578125000, 4.692493439
38, 1.593750000, 5.065765381
39, 1.609375000, 5.442348480
40, 1.625000000, 5.822265625
41, 1.640625000, 6.205539703
42, 1.656250000, 6.592193604
43, 1.671875000, 6.982250214
44, 1.687500000, 7.375732422
45, 1.703125000, 7.772663116
46, 1.718750000, 8.173065186
47, 1.734375000, 8.576961517
48, 1.750000000, 8.984375000

49, 1.765625000, 9.395328522
50, 1.781250000, 9.809844971
51, 1.796875000, 10.22794724
52, 1.812500000, 10.64965820
53, 1.828125000, 11.07500076
54, 1.843750000, 11.50399780
55, 1.859375000, 11.93667221
56, 1.875000000, 12.37304688
57, 1.890625000, 12.81314468
58, 1.906250000, 13.25698853
59, 1.921875000, 13.70460129
60, 1.937500000, 14.15600586
61, 1.953125000, 14.61122513
62, 1.968750000, 15.07028198
63, 1.984375000, 15.53319931
64, 2., 16. **(7)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

> $m[23]; m[24];$

1.359375000
1.375000000 **(8)**

> $k := 3; en := \frac{k \cdot \ln(10)}{\ln(2)}; evalf(en);$

$$k := 3$$

$$en := \frac{3 \ln(10)}{\ln(2)}$$

9.965784285 **(9)**

> $c7 = \frac{(m[23] + m[24])}{2};$

$$c7 = 1.367187500 \quad \textbf{(10)}$$

>

Ejercicio 2 del método de bisección. El intervalo es (0,1).

> $f := x \rightarrow x^5 - 3 \cdot x^2 + 1;$

$$f := x \rightarrow x^5 - 3\,x^2 + 1 \quad \textbf{(1)}$$

> $f(0);$

$$1 \quad \textbf{(2)}$$

> $f(1);$

$$-1 \quad \textbf{(3)}$$

>

Métodos de Maple.

Hallamos las raíces.

>

> $fsolve(f);$

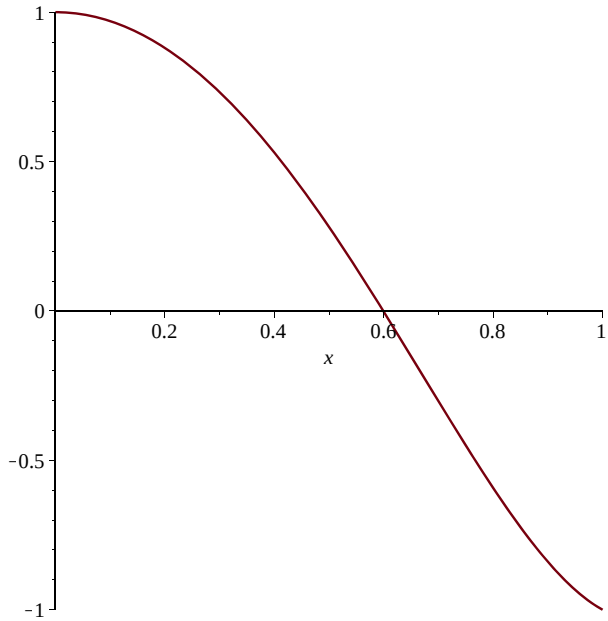
$$-0.5610700072 \quad \textbf{(4)}$$

> $solve(f(x));$

$$RootOf(\_Z^5 - 3\,\_Z^2 + 1, index = 1), RootOf(\_Z^5 - 3\,\_Z^2 + 1, index = 2), RootOf(\_Z^5 - 3\,\_Z^2 + 1, index = 3), RootOf(\_Z^5 - 3\,\_Z^2 + 1, index = 4), RootOf(\_Z^5 - 3\,\_Z^2 + 1, index = 5) \quad \textbf{(5)}$$

>

> $plot(f(x), x = 0..1);$



>

>

> $a := 0; b := 1; n := 7; h := \frac{(b-a)}{2^{n-1}};$

$$a := 0$$
$$b := 1$$
$$n := 7$$
$$h := \frac{1}{64} \quad \textbf{(6)}$$

>

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 0.01562500000, 0.9992675791
2, 0.03125000000, 0.9970703423
3, 0.04687500000, 0.9934084294
4, 0.06250000000, 0.9882822037
5, 0.07812500000, 0.9816923635
6, 0.09375000000, 0.9736400545

7, 0.1093750000, 0.9641269809
8, 0.1250000000, 0.9531555176
9, 0.1406250000, 0.9407288218
10, 0.1562500000, 0.9268509448
11, 0.1718750000, 0.9115269436
12, 0.1875000000, 0.8947629929
13, 0.2031250000, 0.8765664967
14, 0.2187500000, 0.8569462001
15, 0.2343750000, 0.8359123012
16, 0.2500000000, 0.8134765625
17, 0.2656250000, 0.7896524230
18, 0.2812500000, 0.7644551098
19, 0.2968750000, 0.7379017500
20, 0.3125000000, 0.7100114822
21, 0.3281250000, 0.6808055686
22, 0.3437500000, 0.6503075063
23, 0.3593750000, 0.6185431397
24, 0.3750000000, 0.5855407715
25, 0.3906250000, 0.5513312751
26, 0.4062500000, 0.5159482062
27, 0.4218750000, 0.4794279141
28, 0.4375000000, 0.4418096542
29, 0.4531250000, 0.4031356992
30, 0.4687500000, 0.3634514511
31, 0.4843750000, 0.3228055527
32, 0.5000000000, 0.2812500000
33, 0.5156250000, 0.2388402531
34, 0.5312500000, 0.1956353486
35, 0.5468750000, 0.1516980110
36, 0.5625000000, 0.1070947647
37, 0.5781250000, 0.06189604569
38, 0.5937500000, 0.01617631316
39, 0.6093750000, −0.02998583857
40, 0.6250000000, −0.07650756836
41, 0.6406250000, −0.1233016765
42, 0.6562500000, −0.1702764928
43, 0.6718750000, −0.2173357652
44, 0.6875000000, −0.2643785477
45, 0.7031250000, −0.3112990884
46, 0.7187500000, −0.3579867184
47, 0.7343750000, −0.4043257395
48, 0.7500000000, −0.4501953125

49, 0.7656250000, −0.4954693457
50, 0.7812500000, −0.5400163829
51, 0.7968750000, −0.5836994918
52, 0.8125000000, −0.6263761520
53, 0.8281250000, −0.6678981436
54, 0.8437500000, −0.7081114352
55, 0.8593750000, −0.7468560720
56, 0.8750000000, −0.7839660645
57, 0.8906250000, −0.8192692762
58, 0.9062500000, −0.8525873125
59, 0.9218750000, −0.8837354081
60, 0.9375000000, −0.9125223160
61, 0.9531250000, −0.9387501953
62, 0.9687500000, −0.9622144997
63, 0.9843750000, −0.9827038655
64, 1., −1. **(7)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

> $m[38]; m[39];$

0.5937500000
0.6093750000 **(8)**

> $k := 3; en := \frac{k \cdot \ln(10)}{\ln(2)}; evalf(en);$

$$k := 3$$
$$en := \frac{3 \ln(10)}{\ln(2)}$$
9.965784285 **(9)**

> $c7 := \frac{(m[38] + m[39])}{2};$

$$c7 := 0.6015625000 \quad \textbf{(10)}$$

> $f(c7);$

−0.006854537 **(11)**

> $evalf(\%);$

−0.006854537 **(12)**

> $evalf\left(\frac{38}{2^6}\right);$

0.5937500000 **(13)**

>

Ejercicio 3 del método de bisección. Se buscan raíces de 3x-e^x.

> $f := x \rightarrow 3 \cdot x - \exp(x);$

$$f := x \rightarrow 3\,x - e^{x} \quad \textbf{(1)}$$

> $f(0);$

$$-1 \quad \textbf{(2)}$$

> $evalf(f(1));$

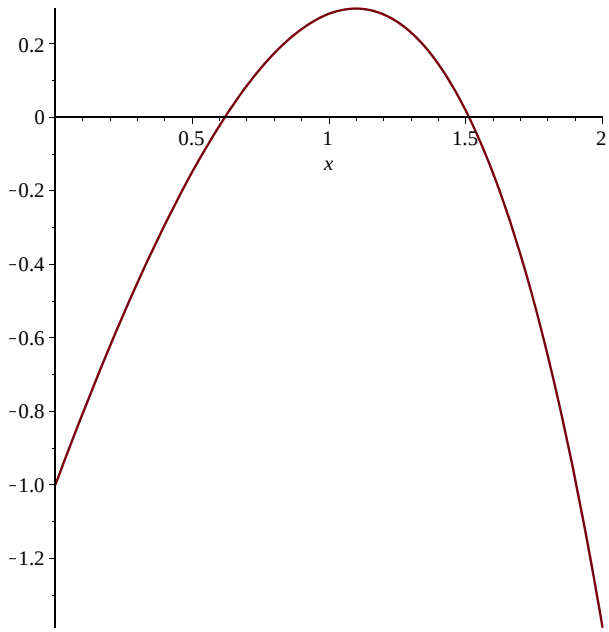
$$0.281718172 \quad \textbf{(3)}$$

> $evalf(f(2));$

$$-1.389056099 \quad \textbf{(4)}$$

>

> $plot(f(x), x = 0..2);$



> $a := 0;\ b := 1;\ n := 7;\ h := \frac{(b-a)}{2^{n-1}};$

$$a := 0$$
$$b := 1$$
$$n := 7$$
$$h := \frac{1}{64} \tag{5}$$

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 0.01562500000, −0.9688727090
2, 0.03125000000, −0.9379934070
3, 0.04687500000, −0.9073660020
4, 0.06250000000, −0.8769944590
5, 0.07812500000, −0.8468828070
6, 0.09375000000, −0.8170351400
7, 0.1093750000, −0.7874556150
8, 0.1250000000, −0.7581484530
9, 0.1406250000, −0.7291179450
10, 0.1562500000, −0.7003684460
11, 0.1718750000, −0.6719043830
12, 0.1875000000, −0.6437302490
13, 0.2031250000, −0.6158506120
14, 0.2187500000, −0.5882701080
15, 0.2343750000, −0.5609934480
16, 0.2500000000, −0.5340254170
17, 0.2656250000, −0.5073708750
18, 0.2812500000, −0.4810347590
19, 0.2968750000, −0.4550220830
20, 0.3125000000, −0.4293379410
21, 0.3281250000, −0.4039875070
22, 0.3437500000, −0.378976035
23, 0.3593750000, −0.354308864
24, 0.3750000000, −0.329991415
25, 0.3906250000, −0.306029195
26, 0.4062500000, −0.282427800
27, 0.4218750000, −0.259192911
28, 0.4375000000, −0.236330299
29, 0.4531250000, −0.213845827
30, 0.4687500000, −0.191745450
31, 0.4843750000, −0.170035217
32, 0.5000000000, −0.148721271
33, 0.5156250000, −0.127809853
34, 0.5312500000, −0.107307302
35, 0.5468750000, −0.087220057

36, 0.5625000000, −0.067554657
37, 0.5781250000, −0.048317746
38, 0.5937500000, −0.029516072
39, 0.6093750000, −0.011156489
40, 0.6250000000, 0.006754043
41, 0.6406250000, 0.024208450
42, 0.6562500000, 0.041199550
43, 0.6718750000, 0.057720047
44, 0.6875000000, 0.073762530
45, 0.7031250000, 0.089319472
46, 0.7187500000, 0.104383227
47, 0.7343750000, 0.118946027
48, 0.7500000000, 0.132999983
49, 0.7656250000, 0.146537084
50, 0.7812500000, 0.159549189
51, 0.7968750000, 0.172028031
52, 0.8125000000, 0.183965213
53, 0.8281250000, 0.195352204
54, 0.8437500000, 0.206180340
55, 0.8593750000, 0.216440820
56, 0.8750000000, 0.226124706
57, 0.8906250000, 0.235222917
58, 0.9062500000, 0.243726230
59, 0.9218750000, 0.251625277
60, 0.9375000000, 0.258910542
61, 0.9531250000, 0.265572359
62, 0.9687500000, 0.271600911
63, 0.9843750000, 0.276986225
64, 1., 0.281718172 **(6)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

> $m[39]$; $m[40]$;

0.6093750000
0.6250000000 **(7)**

> $k := 3$; $en := \frac{k \cdot \ln(10)}{\ln(2)}$; $evalf(en)$;

$$k := 3$$

$$en := \frac{3 \ln(10)}{\ln(2)}$$

9.965784285 **(8)**

> $c7 := \frac{(m[39] + m[40])}{2}$;

$$c7 := 0.6171875000 \quad \textbf{(9)}$$

> $f(c7)$;

$$-0.002144652 \quad (10)$$

> $evalf(\%);$

$$-0.006854537 \quad (11)$$

> $evalf\left(\frac{38}{2^6}\right);$

$$0.5937500000 \quad (12)$$

>

Raíz en el intervalo [1,2].

> $a := 1; b := 2; n := 6; h := \frac{(b-a)}{2^n};$

$$a := 1$$
$$b := 2$$
$$n := 6$$
$$h := \frac{1}{64} \quad (13)$$

> **for** i **from** 1 **to** 2^n **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)));$ **end do**;

1, 1.015625000, 0.285786461
2, 1.031250000, 0.289180644
3, 1.046875000, 0.291890103
4, 1.062500000, 0.293904056
5, 1.078125000, 0.295211550
6, 1.093750000, 0.295801461
7, 1.109375000, 0.295662487
8, 1.125000000, 0.294783151
9, 1.140625000, 0.293151794
10, 1.156250000, 0.290756572
11, 1.171875000, 0.287585458
12, 1.187500000, 0.283626232
13, 1.203125000, 0.278866483
14, 1.218750000, 0.273293606
15, 1.234375000, 0.266894794
16, 1.250000000, 0.259657043
17, 1.265625000, 0.251567139
18, 1.281250000, 0.242611664
19, 1.296875000, 0.232776987
20, 1.312500000, 0.222049262
21, 1.328125000, 0.210414427
22, 1.343750000, 0.197858195
23, 1.359375000, 0.184366058
24, 1.375000000, 0.169923277
25, 1.390625000, 0.154514881

26, 1.406250000, 0.138125665
27, 1.421875000, 0.120740182
28, 1.437500000, 0.102342744
29, 1.453125000, 0.082917414
30, 1.468750000, 0.062448006
31, 1.484375000, 0.040918078
32, 1.500000000, 0.018310930
33, 1.515625000, -0.005390404
34, 1.531250000, -0.030203153
35, 1.546875000, -0.056144820
36, 1.562500000, -0.083233182
37, 1.578125000, -0.111486298
38, 1.593750000, -0.140922509
39, 1.609375000, -0.171560448
40, 1.625000000, -0.203419037
41, 1.640625000, -0.236517500
42, 1.656250000, -0.270875362
43, 1.671875000, -0.306512455
44, 1.687500000, -0.343448925
45, 1.703125000, -0.381705233
46, 1.718750000, -0.421302165
47, 1.734375000, -0.462260831
48, 1.750000000, -0.504602676
49, 1.765625000, -0.548349482
50, 1.781250000, -0.593523374
51, 1.796875000, -0.640146825
52, 1.812500000, -0.688242662
53, 1.828125000, -0.737834072
54, 1.843750000, -0.788944607
55, 1.859375000, -0.841598190
56, 1.875000000, -0.895819120
57, 1.890625000, -0.951632080
58, 1.906250000, -1.009062139
59, 1.921875000, -1.068134764
60, 1.937500000, -1.128875821
61, 1.953125000, -1.191311585
62, 1.968750000, -1.255468742
63, 1.984375000, -1.321374402
64, 2., -1.389056099 **(14)**

> **for** i **from** 1 **to** 2^n **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

> $c7 := \frac{(m[32] + m[33])}{2}$;

(15)

$$c7 := 1.507812500 \quad \textbf{(15)}$$

>

Iteraciones para error menor o igual que 10^{-4}.

> $k := 4;\ en := \frac{k \cdot \ln(10)}{\ln(2)};\ evalf(en);$

$$k := 4$$

$$en := \frac{4 \ln(10)}{\ln(2)}$$

$$13.28771238 \quad \textbf{(16)}$$

>

Ejercicio 4 del método de bisección. Ecuación de Kepler.

> $f := x \rightarrow x - 0.5 \cdot \sin(x) - 0.7;$

$$f := x \rightarrow x + (-1) \cdot 0.5 \sin(x) - 0.7 \quad \textbf{(1)}$$

>

> $evalf(f(1));$

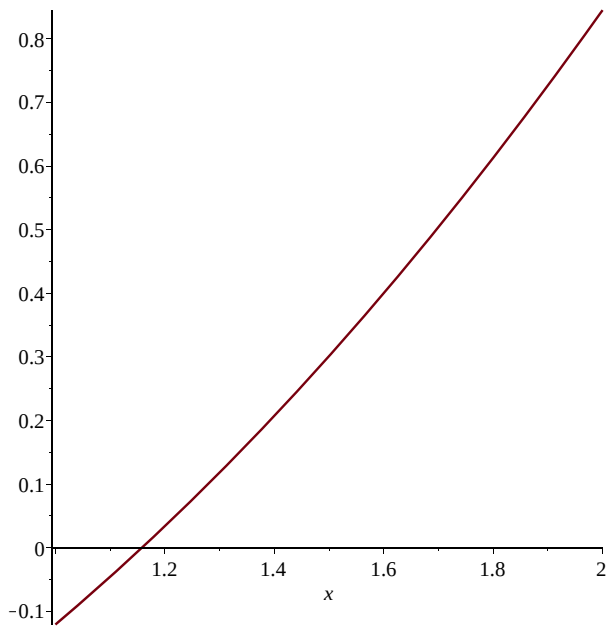
$$-0.1207354924 \quad \textbf{(2)}$$

> $evalf(f(2));$

$$0.8453512866 \quad \textbf{(3)}$$

>

> $plot(f(x), x = 1..2);$



> $a := 1; b := 2; n := 4; h := \frac{(b-a)}{2^{n-1}};$

$$a := 1$$

$$b := 2$$
$$n := 4$$
$$h := \frac{1}{8} \quad \textbf{(4)}$$

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 1.125000000, −0.0261337970
2, 1.250000000, 0.0755076903
3, 1.375000000, 0.1845534715
4, 1.500000000, 0.3012525067
5, 1.625000000, 0.4257343298
6, 1.750000000, 0.5580070266
7, 1.875000000, 0.6979571092
8, 2., 0.8453512866 **(5)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

>

$$9.965784285 \quad \textbf{(6)}$$

> $c7 := \frac{(m[1] + m[2])}{2}$;

$$c7 := 1.187500000 \quad \textbf{(7)}$$

>

Para un error menor que 5*10^{-6} hace falta
n mayor que:

> $\frac{5 \cdot \ln(5)}{\ln(2)} + 6$;

$$\frac{5 \ln(5)}{\ln(2)} + 6 \quad \textbf{(8)}$$

> $evalf(\%)$;

$$17.60964047 \quad \textbf{(9)}$$

>

Ejercicio 5. La raíz 7^{1/4} con valor menor que una centésima.

> $f := x \rightarrow x^4 - 7;$

$$f := x \rightarrow x^4 - 7 \quad \textbf{(1)}$$

>

> $evalf(f(1));$

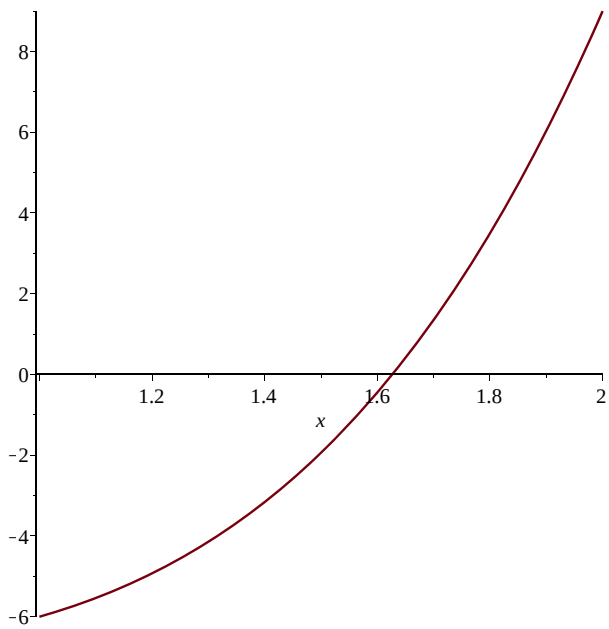
$$-6. \quad \textbf{(2)}$$

> $evalf(f(2));$

$$9. \quad \textbf{(3)}$$

>

> $plot(f(x), x = 1..2);$



> $a := 1; b := 2; n := 4; h := \frac{(b-a)}{2^{n-1}};$

$$a := 1$$

$$b := 2$$
$$n := 4$$
$$h := \frac{1}{8} \quad \textbf{(4)}$$

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 1.125000000, −5.398193359
2, 1.250000000, −4.558593750
3, 1.375000000, −3.425537109
4, 1.500000000, −1.937500000
5, 1.625000000, −0.02709960938
6, 1.750000000, 2.378906250
7, 1.875000000, 5.359619141
8, 2., 9. **(5)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

> $m[5]$; $m[6]$;

1.625000000
1.750000000 **(6)**

>

> $c4 := \frac{(m[5] + m[6])}{2}$;

$$c4 := 1.687500000 \quad \textbf{(7)}$$

>

Ejercicio 6b.

> $f := x \rightarrow \exp(x) - x^2 + 3 \cdot x - 2;$

$$f := x \rightarrow e^x - x^2 + 3\,x - 2 \quad \textbf{(1)}$$

>

> $evalf(f(0));$

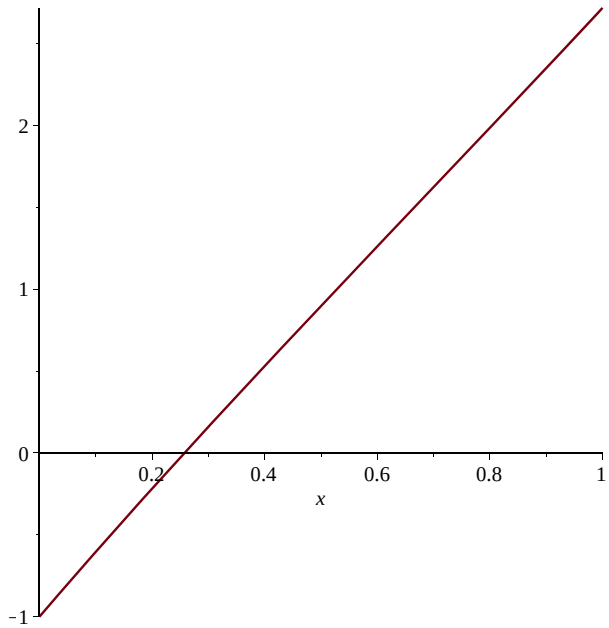
$$-1. \quad \textbf{(2)}$$

> $evalf(f(1));$

$$2.718281828 \quad \textbf{(3)}$$

>

> $plot(f(x), x = 0..1);$



> $a := 0; b := 1; n := 4; h := \frac{(b-a)}{2^{n-1}};$

$$a := 0$$

$$b := 1$$
$$n := 4$$
$$h := \frac{1}{8} \quad \textbf{(4)}$$

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, 0.1250000000, −0.507476547
2, 0.2500000000, −0.028474583
3, 0.3750000000, 0.439366415
4, 0.5000000000, 0.8987212710
5, 0.6250000000, 1.352620957
6, 0.7500000000, 1.804500017
7, 0.8750000000, 2.258250294
8, 1., 2.718281828 **(5)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

>

>

> $c4 := \frac{(m[2] + m[3])}{2}$;

$$c4 := 0.3125000000 \quad \textbf{(6)}$$

>

Ejercicio 6c.

> $f := x \rightarrow 2\,x \cdot \cos(2 \cdot x) - (x+1)^2;$

$$f := x \rightarrow 2\,x\cos(2\,x) - (x+1)^2 \quad \textbf{(1)}$$

>

> $evalf(f(-3));$

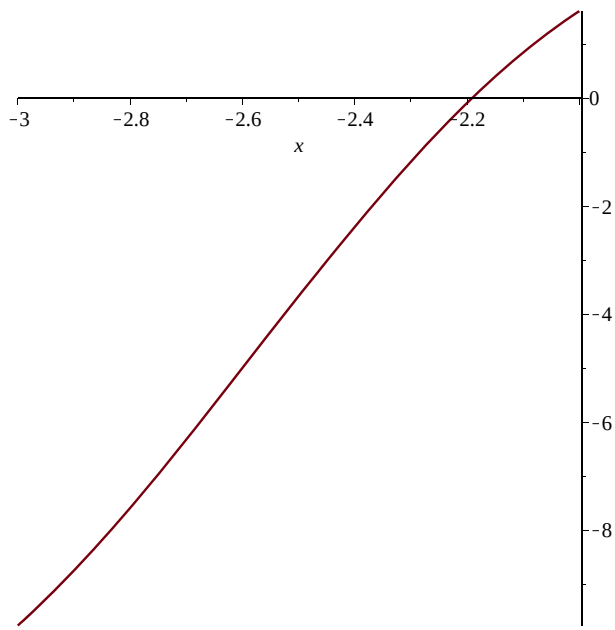
$$-9.761021720 \quad \textbf{(2)}$$

> $evalf(f(-2));$

$$1.614574484 \quad \textbf{(3)}$$

>

> $plot(f(x), x = -3..-2);$



> $a := -3;\ b := -2;\ n := 4;\ h := \dfrac{(b-a)}{2^{n-1}};$

$$a := -3$$

$$b := -2$$
$$n := 4$$
$$h := \frac{1}{8} \quad \textbf{(4)}$$

>

> **for** i **from** 1 **to** 2^{n-1} **do** $print(i, evalf(a + i \cdot h), evalf(f(a + i \cdot h)))$; **end do**;

1, -2.875000000, -8.467481399
2, -2.750000000, -6.960183759
3, -2.625000000, -5.329073755
4, -2.500000000, -3.668310928
5, -2.375000000, -2.069235226
6, -2.250000000, -0.6139189027
7, -2.125000000, 0.630246832
8, -2., 1.614574484 **(5)**

> **for** i **from** 1 **to** 2^{n-1} **do** $m[i] := evalf(a + i \cdot h)$; **end do**:

>

>

> $c4 := \frac{(m[6] + m[7])}{2}$;

$$c4 := -2.187500000 \quad \textbf{(6)}$$

>